WPS7130

Policy Research Working Paper 7130

Incomplete Integration and Contagion of Debt Distress in Economic Unions

Cem Karayalcin
Harun Onder

WORLD BANK GROUP
Macroeconomics and Fiscal Management Global Practice Group
December 2014

Abstract

This paper compares different fiscal integration schemes on the basis of their ability to finance public investments and resilience to debt distress and contagion. Complete integration schemes, where a central authority chooses the level of public investments with productivity-enhancing externalities across different jurisdictions, are shown to be superior to incomplete integration schemes, where member governments choose public investments unilaterally. As a result, equilibrium income is greater for citizens of member states under a complete integration scheme. Moreover, complete integration schemes are shown to be more resilient to idiosyncratic shocks and more effective in limiting contagion of debt distress. This is mainly because the central authority can credibly borrow more without risking default than member states taken together can and it can "transfer resilience" across them if needed. These findings inform discussions on structural aspects of secular stagnation in Europe by emphasizing a potential challenge in the institutional design of fiscal responsibilities.

This paper is a product of the Macroeconomics and Fiscal Management Global Practice Group. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The authors may be contacted at honder@worldbank.org.

Incomplete Integration and Contagion of Debt Distress in Economic Unions*

Cem Karayalcin and Harun Onder†

Key Words: Sovereign Debt, Contagion, Integration, Federalism

JEL Codes: H63, H77, E23

*We would like to thank Aart Kraay, Tito Cordella, Fernando L. Hernandez, and Alvaro Manoel for useful comments provided for an earlier version of this paper.

†Cem Karayalcin, Professor, Department of Economics, Florida International University, E-Mail: karayalc@fiu.edu; Harun Onder, Economist, Economic Advisory Team, The Global Practice for Macroeconomics and Fiscal Management, The World Bank, E-Mail: honder@worldbank.org.

1 Introduction

Years after the beginning of the Great Recession in 2008, sluggish economic growth remains a major challenge in advanced economies, especially in Europe. Policies to revive the growth rates seem to have failed despite the near-zero interest rates for a prolonged period of time; and short and medium-term forecasts for recovery are more pessimistic in 2014 than a few years ago (IMF, 2014). Recently, many economists have expressed concerns that this may not be the typical hangover from a financial crisis. In late 2013, the former US Secretary of Treasury Lawrence Summers suggested that it may not be possible to achieve full employment, satisfactory growth, and financial stability simultaneously simply through the means of monetary policy. Thus, a "secular stagnation", a term first introduced by Alvin Hansen back in 1938, or "the new mediocre" as described by the Chief of the International Monetary Fund (IMF), Christine Lagarde, could be the name of anemic growth rates in the years to come.

Although Keynesian aspects constitute an important share of the discussions around secular stagnation, structural factors have also received considerable attention. Gordon (2012) and Gordon (2014) suggest a number of headwinds, including aging demography, rising inequality, and public debt, that are likely to lead to reduced growth in the future. Glaeser (2014), on the other hand, argues that even though the speed of technological innovations has not changed, they may have fewer beneficiaries in recent decades. As a result, the impact of innovations on the productive capacity of the economy could be limited compared to previous decades.

The emphasis on both short-term and structural factors behind weak growth rates is especially noticeable in policy discussions around the recovery in Europe. The IMF has recently called for urgent structural reforms and a surge in infrastructure investments to both boost the demand in the short-term and strengthen potential growth in the long-term (IMF 2014). Similarly, European Union (EU) finance ministers endorsed plans to leverage hundreds of billions of euros to finance new infrastructure projects in September 2014. Further, a number of economists have pointed to the fact that the slow-down in European growth began long before the crisis, and have emphasized the need for structural reforms in Europe where secular stagnation is most likely to become reality (Buiter et.al. 2014, and Jimeno et.al. 2014). These observations imply that the debt distress in a number of European economies following the American sub-prime mortgage crisis, and the associated fear of contagion which eventually led to cuts in public expenditures in the spirit of austerity, could have stemmed from deep structural problems, which contribute to the risk of secular stagnation in the region.

A particularly interesting angle that is relevant for our discussions in this paper was provided by the Finance Minister of Poland, Mr. Szczurek.[1] He suggested a European Fund for Investment to be backed up by all EU member states just as in the case of the European Stability Mechanism (ESM). This fund would use market mechanisms to invest directly in public infrastructure projects across the Union. Despite the lack of consensus among member states, however, he believes that this is a case where the EU members can find mutually beneficial solutions to their shared problems.

The call for a centralized fund to coordinate investments in the EU raises two questions that motivate the analysis in this paper:

- *First, what are the factors that have led the individual member states to hold back from undertaking such investments unilaterally?*
- *Second, is the Union as a whole in a better position to implement investments as opposed to individual member states?*

In answering these questions, this paper suggests that institutional factors may also contribute to secular stagnation and instability in economic unions. In particular, we show that the degree of fiscal integration has important consequences for long-term income levels and the resilience of the union to debt distress that can be triggered by productivity shocks. Complete integration schemes, where a centralized authority holds the mandate to make the decisions for shared economic issues, are shown to perform better than incomplete integration schemes, where member states decide on economic actions unilaterally even if those would have consequences for other members.

To fix these ideas, we establish a simple model comprising an economic union with two member states.[2] Each of the two economies produces a composite good employing both private and public capital. An important characteristic of the model is that public capital in one economy leads to productivity enhancing spillovers in the other member of the union. In practice, this would be the case if investments in, say, transportation infrastructure in one region reduce production costs in a neighboring jurisdiction by improving its connectivity as well.[3] These spillovers play an important role for our results.

[1]The speech was delivered at the annual Bruegel dinner. For the transcript, see http://www.voxeu.org/article/investing-europe-s-future.

[2]We use the words economy and country interchangeably to denote the nations that are members of the economic union. Similarly, the governments of each nation are denoted by nation states, member states or national governments. The supra-national level of government is named the central government, referring to a case with complete integration, *i.e.* federalism.

[3]Cohen et.al. (2004) show that there are significant interstate spillovers of public infrastructure investments by using US manufacturing sector between 1982 and 1996. They conclude that most of this spillovers stem from savings from transportation costs.

In the case of an incomplete integration scheme, such as the European Union, member states use their tax proceeds and the funds raised in international markets in the form of external debt to invest in public capital. The decision-making problem faced by the national government is that of maximizing the welfare of its constituents by choosing the optimal level of investment. While solving this optimization problem, however, national governments do not internalize the productivity spillovers across jurisdictions.

In comparison, in the case of a complete integration scheme, for example federations like the United States, spillover generating investments in each economy are chosen by the central (supranational in the case of the EU) government. The difference between the two integration schemes mainly lies in the fact that the central government internalizes the spillovers across the member economies, whereas national governments do not. Therefore, perceived returns on a given investment are greater to the central government than to a national government. As a result, the optimal levels of investment in both economies are greater under a complete integration scheme than an incomplete one.

In addition to choosing the optimal investment levels, the governments can also choose between repaying their debt or defaulting. In the case of repayment, the amortization includes the principal and a pre-determined interest payment. In comparison, default triggers a punishment phase, where the lenders can capture a fixed share of output in the economy on which the government is defined, *i.e.,* only the member economy in the case of an incomplete integration and the entire union in the case of a complete integration. A large enough loan that makes the repayment more costly compared to punishment would, then, trigger a default. Moreover, anticipating this default, the borrowing government re-optimizes the level of investment so as to maximize the welfare of its constituents net of punishments accordingly.

Our first result builds on this observation: the amount of foreign funds that can be raised credibly for investment is greater in complete integration schemes. This is shown by comparing the incentives to repay the debt under the two integration schemes. Higher public investment, thus output, in complete integration schemes leads to a higher amount foreign lenders can capture in case of a default (since foreign creditors are able to extract an exogenously-determined share of the output). In comparison, the cost of repaying the debt remains the same between the two integration schemes if interest rates were identical in the beginning. Therefore, the creditors would lend more to a centralized union, whose default is more costly, and therefore, less likely. As a result, the equilibrium income is greater under a complete integration scheme partially because the central government can credibly commit to repay a greater amount of loan.

Next, we investigate the impact of productivity shocks on investment and debt re-

payment decisions under the two schemes. Suppose that an unanticipated negative productivity shock hits one of the member economies, leading to a reduction in its potential income.[4] When the integration is incomplete, a large shock could lead to a default in the economy that is subjected to the shock. In addition, when investments are strategic complements, the policy maker in the economy that experiences the shock would decrease its investments. This reduction, in turn, would decrease the returns to investment in the other member economy, leading to a reduction in investments there as well. If this effect is large enough, the second economy could also default even if there is no shock there. Therefore, this contagion effect could lead to a region-wide reduction in incomes even with idiosyncratic shocks when the integration is incomplete.

In comparison, our simulations show that both a default in the economy that is subject to shocks and contagion of debt distress could be prevented in a complete integration scheme. Starting from identical conditions, *i.e.,* the same level of total debt and economic fundamentals, there exists a range of productivity shocks that lead to default and contagion under incomplete integration, but not under complete integration. This is mainly because the debt ceiling faced by a central government is greater than that of the national governments in a union. When an idiosyncratic shock hits one of the member economies, the central government can "carry resilience" between the regions as there is no partial default, and punishment would be on the basis of total output of the region. Note, however, that the central government may also choose to default if the shock is large enough. Nevertheless, the minimum level of shock that triggers a default in this case is strictly greater than the one that triggers default and contagion in the case of incomplete integration.

The analysis in this paper brings together two strands of literature. The first one concerns sovereign debt under self-enforcing contracts. Our interpretation of the equilibrium in debt contracts is similar to those of the classic theory of sovereign debt, where default triggers a punishment in the form of trade sanctions or appropriation of assets. Sachs and Daniel Cohen (1982), Bulow and Rogoff (1989), and Fernandez and Rosenthal (1990) characterize the threat of such direct punishments to motivate the existence of debt in a long-term relationship. More recent studies highlight the implicit costs driven by sovereign defaults. Alfaro and Kanczuk (2005) emphasize the costs of default taking the form of higher interest rates and losses of output.[5] Mendoza and Yue (2008) show that default episodes are associated with sharp fluctuations in output that are potentially driven by

[4]This would be an environment where the country has a sizable debt stock prior to the shock.

[5]Sandleris (2008) develops a model in which lending to sovereigns can emerge in a single-shot game even in the absence of direct sanctions.

inefficient labor reallocation and sudden total factor productivity drops. For our purposes the former approach is preferred as it imposes less structure on the design of an economic environment. Panizza, Sturzenegger, and Zettelmeyer (2009) provide an excellent review of different approaches.

The second strand of the literature that is pertinent focuses on the provision of public goods in fiscal federalism. The classical approach formulated in Oates (1972) concerns the issue of the provision of local public goods that may have beneficial spillovers to citizens in other jurisdictions. The underlying question in this approach is whether a centralized system in which a central authority makes the spending decisions using resources made available from general revenues would do better than a decentralized system in which local governments make the choices using local taxes to finance them. The advantage of the centralized system is that it internalizes spillovers, but it may use a "one size fits all" approach that may not do justice to local needs.[6] Besley and Coate (2003) point out that the assumption that centralization implies uniformity, which plays a crucial role in the trade-off between the centralized and decentralized systems in this literature, is hard to justify theoretically and empirically. To the extent that costs are shared in a centralized system but the allocation of public goods is not uniform, there will arise a conflict of interest between citizens in different jurisdictions. Besley and Coate then show that depending on how a legislature that makes the choices for the centralized system behaves, misallocation across districts may arise. In this paper, we allow optimal allocation of public infrastructure investment across jurisdictions, which may or may not be uniform when jurisdictions differ. In the process of doing so, we emphasize the role of spillovers but for reasons other than the ones suggested in the existing literature as we focus on the interplay of two intertemporal mechanisms (the consumption-smoothing and rate-of-return) in the relative performance of the centralized and decentralized systems for citizens in different jurisdictions. In addition, our analysis formalizes the nexus between public infrastructure investment and inter-jurisdictional differences for the choice between centralized and decentralized regimes as discussed in Rodden (2005), who hypothesizes that a decentralized system with a weak center would allow wealthy regions to provide public infrastructure to their constituents and poorer regions would therefore prefer more centralized systems that would enable them to capture some of the wealth generated in the richer core regions.

The remainder of the paper is organized as follows. Section 2 sets up the general model. Sections 3 and 4 characterize our analytical findings under incomplete and com-

[6]See also Oates (1999) and (2005) for two useful survey papers that reflect the evolution of the literature over time on this.

plete integration frameworks, respectively. The analysis in these sections begins with a case with no international borrowing, and then extends the framework to include the possibility of international debt. Section 5 introduces the productivity shocks under both integration schemes. Results in this section are driven by using numerical simulations as analytical solutions do not exist. Finally, section 6 offers some concluding remarks.

2 A General Model

We consider the simplest framework that can capture our main points, which consists of an economic union with two members: country 1 and country 2. Each country is inhabited by households/firms (whose mass is normalized to unity) that live for two periods. In the first period agents in country i receive an endowment income y_{1i} which can be used for both consumption and investment purposes. In the second period of their lives they have access to the following production technology to produce a good y_{2i},

$$y_{2i} = A_i k_i^{\alpha} (g_i^{\beta} g_j^{\gamma}) \tag{1}$$

where A, k, and g denote a productivity parameter, a private input (to be interpreted as either produced or non-produced in what follows), and public goods, respectively. The formulation allows for spillovers across jurisdictions such that public goods provided by country j affect output in country i. We define γ such that $\gamma = \theta\beta$ with $\theta \leq 1$ to capture the notion that these spillovers are less than one-to-one across economies.[7] Following Besley and Coate (2003), we abstract from mobility of labor and capital across jurisdictions.

Public investment goods are produced using the following production technology

$$g_i = \Omega_i z_i^{\mu}, \tag{2}$$

where Ω and z denote the productivity parameter and the level of input into the public good production process in each country. We suppose that $\mu \geq 0$ but not necessarily below unity to capture any returns to scale in the production process.

Households in both economies maximize lifetime utility

$$U_i = u(c_{1i}) + \xi u(c_{2i}) \tag{3}$$

[7] Among others, Cohen and Paul (2004) provide evidence for the existence of such inter-state productivity spillovers. Using the manufacturing series in US States between 1982 and 1996, they show that infrastructure investments in neighboring states increase the value of own-state infrastructure investments a well as reducing the manufacturing costs potentially through cost savings in transportation.

(where ξ is the discount factor) subject to the budget constraint

$$c_{1i} + R^{-1}c_{2i} = (1-t_{1i})(1-t^c_{1i})(y_{1i}-k_i) + R^{-1}(1-t_{2i})(1-t^c_{2i})[y_{2i}+(1-\delta_k)k_i+(1-\delta)z_{2i}] \tag{4}$$

with R, t_{ji}, t^c_{ji}, δ_k and δ denoting the gross rate of interest, the proportional income tax rates imposed by the national government (NG) and the central government (CG) of the economic union in period j in country i and the rates of depreciation of k_i and z_{2i}.

NGs finance the cost of providing the public good potentially from three sources: (1) tax revenues accruing from local taxes, (2) foreign borrowing (b_i), and (3) transfers from the central government (τ_i):

$$z_{1i} = t_{1i}(1-t^c_{1i})y_{1i} + b_{1i} + \tau_{1i} \tag{5}$$

$$z_{2i} = t_{2i}(1-t^c_{2i})y_{2i} - \kappa_i + \tau_{2i}. \tag{6}$$

where κ is payments to international lenders in the second period.

The central government (CG) of the union collects taxes from households/firms and uses the proceeds to finance the provision of public goods either directly or indirectly through transfers to NGs.

A key aspect of the model concerns the identity of the government that makes the public investment decisions. In particular, we are interested in comparing optimal investments at the national level with the ones at the Union level. We will begin by solving the equilibrium conditions for each of these cases in the next section. We will first introduce a simple version of the two-period model for which some analytical results can be obtained. After using this simplified model to compare the results for the NGs and the CG for the closed economy case, we will go back to the more general case on which our simulation results are based and introduce international borrowing.

3 Unification with Incomplete Integration

We call integration "incomplete" when the central government of the union has no formal power to influence public investment decisions at the national level, although those investments may have cross-jurisdiction externalities, *i.e.* productivity in one economy depends on the investments in the other. This could be considered as a case where the Oates' perfect correspondence principle is violated in the formation of the union.[8] In this

[8] A perfect correspondence principle exists when the jurisdiction that provides the public good perfectly matches the constituents who consume it. In the case of economic integration, the type of investments

case, member states implement unilaterally optimal levels of investments. To simplify the analysis, we will first consider the case where member states are symmetric and the central government plays no active role:

$$t^c = \tau = 0. \tag{7}$$

Moreover, we assume that the private input used in second-period production is a non-produced and inelastically supplied input, say, labor.[9]

3.1 National Investment Policies in the Absence of Debt Issuance

We first analyze investment policies where the member countries have no access to global financial markets. In this case, NGs tax their residents in the first period to finance investment in public infrastructure which bears fruit in the second period. Thus for the residents of a given jurisdiction i we have $c_{1i} = y_{1i} - t_{1i}y_{1i} = y_{1i} - z_i$ and $c_{2i} = y_{2i} + (1-\delta)z_i$, where y_{2i} is given by (1) and (2). Using logarithmic period utility functions $u(c_i) = \ln(c_i)$, maximization of (3) subject to these budget constraints yields

$$y_{2i} + (1-\delta)z_i = \xi[(1-\delta) + \rho_i](y_{1i} - z_i),\ \rho_i \equiv \partial y_{2i}/\partial z_i. \tag{8}$$

Note that, although the central government does not have a role in this process, spillovers across the countries link the decision-making problems of the separate NGs with regard to public investment. Thus, the problem is inherently strategic. Equation (8) yields the best-response function z, that is, the unilaterally optimal z_i given z_j and the parameters

$$z_i = z(z_j) \tag{9}$$

with

$$\frac{\partial z_i}{\partial z_j} \equiv z'(z_j) = \frac{\xi(y_{1i} - z_i)\rho_{ij} - y_{ij}}{(1+\xi)[\rho_i + (1-\delta)] - \xi(y_{1i} - z_i)\rho_{ii}} \lesseqgtr 0 \tag{10}$$

where

$$\rho_{ij} \equiv \frac{\partial \rho_i}{\partial z_j} > 0,\ y_{ij} \equiv \frac{\partial y_{2i}}{\partial z_j} > 0,\ \textit{where}\ \rho_{ii} \equiv \frac{\partial \rho_i}{\partial z_i} < 0. \tag{11}$$

This leads to our first result that establishes the association between national invest-

which have spillovers across the borders of member states, thus, would required to be undertaken by a central authority.

[9]Below we relax this assumption and allow private agents to accumulate capital. We then solve the model analytically with some further simplifying assumptions to explore the consequences of private capital accumulation for the public provision of infrastructure.

ments in an incomplete integration framework and highlights the crucial role played by intertemporal considerations in determining whether public investments are strategic substitutes or complements.

Result 1: *In a union of symmetric countries with incomplete integration and spillovers, investments of member countries are*
(i) strategic substitutes, $\frac{\partial z_i}{\partial z_j} < 0$, *when* $\mu\beta < 1$, *and*
(ii) strategic complements, $\frac{\partial z_i}{\partial z_j} > 0$, *when* $\mu\beta > 1$.

The proof of this result is provided in the appendix to this paper. Intuitively speaking, a rise in z_j affects z_i through two intertemporal channels: (1) it raises the return on z_i and, thus, encourages more investment, raising z_i; (2) it also increases future output y_{2i}, decreasing current period saving (and investment z_i) for consumption smoothing purposes. If $\mu\beta > 1$, given $\rho_{ij} = \mu\beta(y_{2i}/z_i)$ the first channel dominates, rendering public investments in the two jurisdictions strategic complements. Otherwise, the effects of public infrastructure spillovers from the other region make public investments strategic substitutes.

The crucial role played by the parameters β and μ in determining whether these investments are complements or substitutes can be intuitively understood in the following way. Start with the observation that μ is a measure of the returns to scale in the production of public infrastructure, while β is the elasticity of the output of the private good with respect to public infrastructure. For any given β, higher returns to scale in public infrastructure production, thus, increase the likelihood that public investments become strategic complements. Similarly, for any given μ, higher elasticities of output with respect to public investment do the same.

Finally, note that the intertemporal mechanisms at play here also explain why the effects of shocks to expected future productivity, such as changes in A_i in equation (1) depend on the magnitude of $\mu\beta$. Again, say an increase in A_i raises the return to public investment but also reduces current saving via the consumption-smoothing channel. Which of these two effects dominates depends on whether $\mu\beta$ exceeds unity or not.

3.2 National Investment Policies with Debt

In this section, we allow member countries to have access to the international capital market and consider borrowing to potentially finance public investments. An important issue that arises in this case is the level of borrowing that is available to a government

that typically cannot commit either to repay or to invest at a predetermined level.

In general, the ability of foreign lenders to punish a sovereign borrower is limited. Therefore, the absence of a commitment to repay implies that whether the country meets its debt obligations or defaults depends on the country's willingness to pay rather than its ability to pay. In the rest of the paper, following the literature on sovereign risk, we will focus on the case where foreign lenders can impose (limited) direct punishments on defaulting borrowers. Specifically, we assume that creditors can impose sanctions proportional to the country's output. As a result, the country's repayment can never exceed the cost of the sanctions. Further, the absence of a commitment to an investment strategy implies that the country has the option of enjoying a first-period consumption binge and investing little. This lowers second-period output and reduces the cost of sanctions, weakening their power to deter default. Thus, potential competitive creditors have to determine how much they can lend to ensure a sufficient level of public investment that makes default more costly than repayment. Thus, once the credit is extended, the borrower decides either to amortize the debt at the contracted rate or to default and pay the punishment, which we call debt distress.

To see what is involved, focus on the first member of the union. The government in this country maximizes the lifetime utility (equation (3)) of its constituents subject to (1) in the second period, (5) and (6). In addition, it collects taxes and borrows internationally in period 1 to invest z that yields the consumption good in period 2.

The payment to international lenders in the second period takes the form

$$\kappa = \begin{cases} Rb_i & \texttt{if non} - \texttt{default} \\ \eta(y_{i2} + (1-\delta)z_i) & \texttt{if default} \end{cases} \tag{12}$$

where η is the fraction of second period income lenders can capture in the case of a default.

Note that the decision as to whether to default or to pay back the debt is made when the payment is due in the second period. However, in a two period model, the borrowing government can already incorporate this option while deciding on the level of investment in the first period by using backward induction. Thus, the optimal investment decision is required to be subgame perfect.

To see this, note that each member government maximizes the total utility of their constituencies given the level of debt, b_i, and the other country's level of public investment z_j:

$$U_i(z_i; b_i, z_j) = \max \left\{ U_i^{ND}(z_i; b_i, z_j), U_i^{D}(z_i; b_i, z_j) \right\}$$

where

$$U_i^{ND}(z_i; b_i, z_j) = \max \left\{ \ln(y_{i1} + b_i - z_i) + \xi \ln(y_{i2} + (1-\delta)z_i - Rb_i) \right\} \tag{13}$$

$$U_i^{D}(z_i; b_i, z_j) = \max \left\{ \ln(y_{i1} + b_i - z_i) + \xi \ln(1-\eta)[y_{i2} + (1-\delta)z_i] \right\}. \tag{14}$$

In order to see how much investment will be undertaken for a given level of lending under the two scenarios, we need to find the optimal levels of investment for both cases. The first-order conditions for the two problems faced by a potential borrowing government are given by

$$(1+\tilde{r}_i)\xi(y_{1i} + b_i - z_i) - (y_{2i} + (1-\delta)z_i - Rb_i) = 0 \Rightarrow z_i^{ND} = f(b_i; z_j) \tag{15}$$

$$(1+\tilde{r}_i)\xi(y_{1i} + b_i - z_i) - (1-\eta)(y_{2i} + (1-\delta)z_i) = 0 \Rightarrow z_i^{D} = h(b; z_j) \tag{16}$$

$$\tilde{r} \equiv dy_{2i}/dz_i \tag{17}$$

Equations (15) and (16) yield the levels of public investment under default, z_i^D, versus repayment of the loan, z_i^{ND}, as functions of b_i (and the expected level of z_j). As it is difficult to characterize these conditions analytically in the general case, it is useful to first focus on the simple case of full depreciation. Here we start by setting $\delta = 1$ in equations (12) to (16). We then note that in the case of an NG that takes public investment by the other as given, the rate of return it faces (given the production function we adopt) is

$$\tilde{r}^{NG} = \mu\beta y_2/z. \tag{18}$$

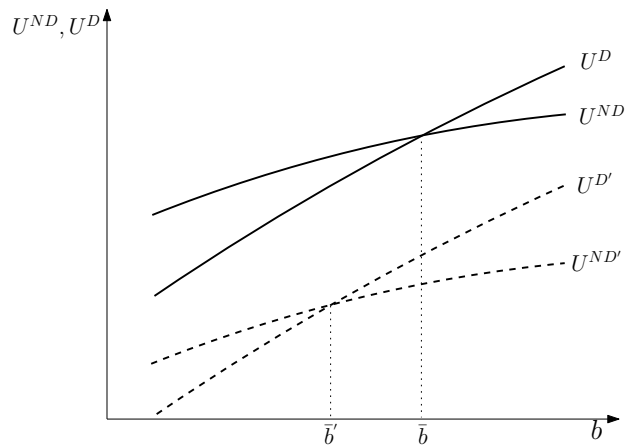
(15), (16), and (18) allow us to solve the difference between optimal level of public investment by an NG that plans to pay back versus default as

$$z_i^{ND} - z_i^{D} = \frac{\mu\beta R b_i}{\tilde{r}(1+\xi\mu\beta)} > 0, \tag{19}$$

which indicates that an NG that plans to default will choose to invest less than an NG that plans to make a full repayment of its foreign debt.

It is also straightforward to show in the general case with $\delta \neq 1$ that there exists a debt threshold beyond which the government would choose to default. To see this note first

Figure 1: Incentive Compatibility in Debt Repayment



that

$$\frac{\partial U^D}{\partial b} - \frac{\partial U^{ND}}{\partial b} = \frac{\xi R}{(y_{2i} + (1-\delta)z_i - Rb_i)} > 0. \tag{20}$$

Note that at $b = 0$

$$U^{ND} - U^D = \xi\eta \ln(y_{2i} + (1-\delta)z_i) > 0 \tag{21}$$

Thus, by the continuity of the U^{ND} and U^D these two curves will intersect at some debt limit $\bar{b}$ that makes the borrowing government indifferent between default and full repayment of debt.[10] A useful way to interpret $\bar{b}$ is to note that this is the maximum level of credit that can be extended to the NG without triggering debt distress in the absence of expected future shocks. We will come back to this interpretation in the following sections. Solid curves in Figure (1) depict the determination of $\bar{b}_i$ given an expected level of z_j.

So far, we have focused on decision making by an NG that expects a certain level of public investment by other NG(s) in the Union. For non-default to be a Nash equilibrium while these expectations are fulfilled, it has to be the case that

$$U_i(z_i^{ND}, z_j^k) \geq U_i(z_i^D, z_j^k),\ k = D, ND. \tag{22}$$

In what follows, we will assume that there exists $\bar{b}_i$ that satisfies these conditions given the agents' expectations at period-1 concerning period-2 parameters.

We should also note that our results concerning the strategic substitutability and com-

[10]The inequality in equation (21) holds for realistic values of y_2 and z.

plementarity of the public investments by the NGs discussed as Result 1 continue to hold here in the presence of debt.

4 Complete Integration in a Union

We now turn to the case where the Union in question is fiscally "completely" integrated. We call an integration "complete" when the central government of the union (CG) chooses the level of investment in both regions. Here public investment in either member country is a public good that determines the level of consumption in all member countries simultaneously. Thus, in the spirit of Oates' perfect correspondence principle, the provision of these public goods is undertaken here by the level of government that is defined over the entire population of the union.

4.1 Investment Policies of the Union in the Absence of Debt Issuance

In this case, the central government (CG) collects taxes from households/firms and uses the proceeds to finance the provision of regional public goods either directly or indirectly through transfers to member states. We focus on the investment decisions by the CG, and assume that the member governments play no active role. Thus $t_{si} = 0$ $(s, i \in \{1, 2\})$ and $\tau = 0$.

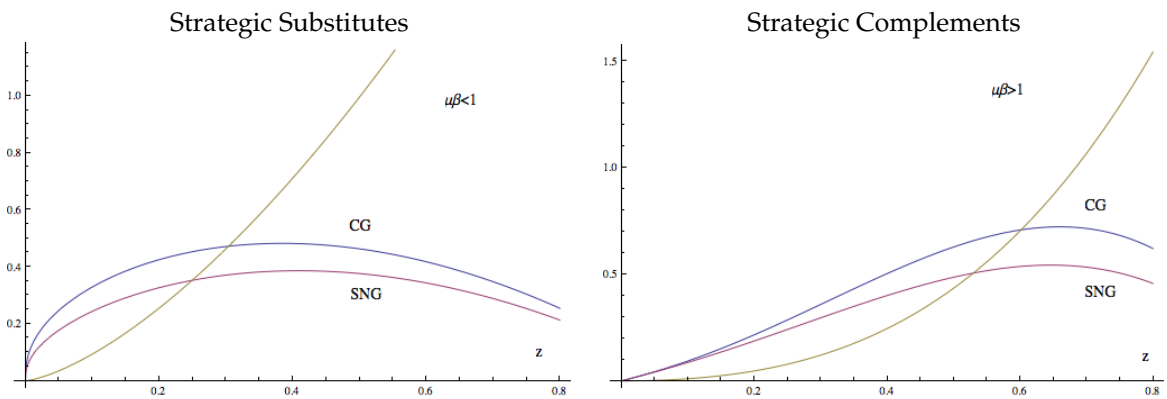
Formally, the CG maximizes the aggregate utility, $\Sigma_{i=1}^{2} U_i$ (see (3) subject to (4)). The first-order conditions for this problem $(i \in \{1, 2\})$ are given by

$$\xi \left\{ \frac{\rho_i^{CG} + (1-\delta)}{y_{2i} + (1-\delta) z_i} + \frac{\mu \beta \theta y_{2j} / z_i}{y_{2j} + (1-\delta) z_j} \right\} = \frac{2}{\Sigma y_{1i} - \Sigma z_i} \tag{23}$$

It is easy to see that there are two main differences between the optimal decision of the CG and that of the NGs: (1) the former takes into account the externalities generated by infrastructure investments in the two regions, and (2) while the NGs are concerned with the welfare of the residents of their particular jurisdictions, the CG is concerned with the welfare of residents of all jurisdictions of the union. An important implication of this last difference is that the CG here uses the only tool in its arsenal, infrastructure investment, to redistribute resources from rich to poor members.

Analytically, the difference is seen most starkly in the simplest case of $\delta = 1$, where it

Figure 2: Equilibrium Levels of Investments in Centralized and Decentralized Systems



is straightforward to show

$$z^{CG} = \frac{\xi(1+\theta)\mu\beta}{1+\xi(1+\theta)\mu\beta} y_1 > z^{NG} = \frac{\xi\mu\beta}{1+\xi\mu\beta} y_1.$$

Thus, the CG invests more than the NGs with the consequence that the economies will grow faster (that is,y_{2i}'s will be higher) under the CG.

To compare investment decisions of the CG with that of two symmetric NGs in the general case with $\delta \neq 1$, note that the first-order conditions of their respective maximization problems yield

$$y_2 + (1-\delta)z = \xi((y_1 - z)/z)[\pi_i + (1-\delta)z],\ \pi_{CG} = \mu\beta(1+\theta)y_2,\ \pi_{NG} = \mu\beta y_2 \qquad (24)$$

(where we dropped the subscripts for the two economies). The left and right-hand sides of the equations (24) are depicted in figure (2) for the cases of strategic substitutes and complements. These figures show that as the CG takes into account the externality, it invests more than the NGs. This constitutes our next result.

Result 2: *In a union with complete integration of symmetric regions and spillovers*
(i) public investments by the central government of the union are greater than those undertaken by national governments of member countries. Consequently, complete integration generates more income than incomplete integration.
(ii) This result holds when public investments in separate regions are strategic complements or strategic substitutes.

This finding is broadly in line with the classical result known as the *Oates' Decentral-*

ization Theorem: without spillovers, a decentralized system is superior (Besley and Coate, 2003). However, with spillovers and symmetric regions a centralized system delivers better outcomes. Since changes in the parameters of the model affect the decisions of the CG in the same qualitative way they do the NGs, we leave the analysis of such changes to the interested reader and turn to the discussion debt issuance.

4.2 Investment Policies of the Union with Foreign Debt

First, note that under the assumption that the union remains a small economy, the CG solves a similar problem as the two NGs when it is able to contract debt in international capital markets. The difference with the CG is that it takes into account the externalities generated by the investment in public infrastructure one NG undertakes on the agents residing in the jurisdiction of the second NG. The most important consequence of the internalization by the CG of the externalities involved is that in equilibrium the CG would receive a higher rate of return.

Formally, the CG again maximizes the aggregate utility, $\Sigma_{i=1}^{2} U_i$ (see (3) subject to (4)) of its constituents. In addition, it collects taxes and borrows internationally in period 1 to finance investments z_i. In period 2, the CG imposes taxes to meet its international obligations which take the form

$$\kappa = \begin{cases} Rb^* & \texttt{if non} - \texttt{default} \\ \eta(\sum_{i=1,2} y_{2i}^* + (1-\delta)\sum_{i=1,2} z_i^*) & \texttt{if default} \end{cases} \tag{25}$$

where we used the superscript "*" to denote the values under the centralized solution. As in the decentralized solution, we focus on equilibria with no default on the equilibrium path. Thus, subgame perfection requires that, given the magnitude of debt and investments in each region, the central government of the union cannot default gainfully in the second period $\sum_{i=1,2} U^{ND}(b_i^*) > \sum_{i=1,2} U^D(b_i*)$, where (using the budget constraints)

$$U_i^{ND}(b) = \ln(y_{1i}^* + b_i^* - z_i^*) + \xi \ln(y_{2i}^* + (1-\delta)z_i^* - Rb_i^*) \tag{26}$$

$$U_i^{D}(b) = \ln(y^*{}_{1i} + b_i^* - z_i^*) + \xi \ln(1-\eta)[y^*{}_{2i} + (1-\delta)z_i^*]. \tag{27}$$

Following the same procedure as in Section 3.2, it is straightforward to show that there exists a debt ceiling faced by the CG. To see how this debt ceiling compares with the ones faced by the NG, first note that in a symmetric equilibrium $b_1^* = b_2^* = \frac{b^*}{2}$ and $z_1^* = z_2^*$.

Thus, we can write the above conditions as follows

$$b^* < 2\frac{\eta}{R}\left[y_{2i}^* + (1-\delta)z_i^*\right] \tag{28}$$

Let us denote the maximum level of debt that induces repayment with $\bar{b}^*$. As $z_i^* > z_i$, thus $y_{2i}^* > y_{2i}$, for $i = 1, 2$ by Proposition 2, this observation concludes that there exists an interval $2\bar{b}_1 = 2\bar{b}_2 < b < \bar{b}^*$. Therefore, lending to the central government of a union does not lead to a default, yet the same magnitude of debt triggers debt distress in a union with incomplete integration when shared equally between the regions. We summarize this finding in the following proposition.

Result 3: *In a union with symmetric regions and spillovers, the level of debt that can be accumulated without triggering debt distress is greater if integration is complete.*

Intuitively, for a given interest rate the cost of defaulting is greater in a centralized system. This is mainly because higher returns due to productivity spillovers lead to greater investments under a centralized system, thus to greater second period income. This, in turn, increases the value of punishment, *i.e.* the share of output that could be captured by the creditors, deterring the borrower from default.

5 Debt Dynamics and Contagion of Shocks

Next, we compare the complete and incomplete integration schemes in terms of their resilience against productivity shocks. In particular, we are interested in investigating if there are differences between how these two systems either magnify or alleviate debt distress triggered by productivity shocks.

We begin by emphasizing that the timing of the shock is an important factor. When a productivity shock hits in the second period, the only decision the borrower can make is to choose between defaulting and repaying the debt. However, revelation of information about the shock prior to the investment decision will cause the borrower to reconsider her investment decision. We now turn to these issues.

5.1 Shocks with Irreversible Debt and Investment Decisions

Unexpected falls in productivity lead to debt distress in both complete and incomplete integration schemes if investments are irreversible and borrowing levels are fixed at respective threshold levels before the shock hits. This observation follows from the debt

threshold definitions. However, a more interesting case is when initial debt levels are fixed at a fraction of the respective thresholds. This leaves some slack that could potentially help accommodate productivity shocks without triggering a default. In what follows, we compare the two integration schemes in terms of their resilience to productivity shocks under similar conditions, *i.e.* similar shocks and initial indebtedness.

We begin with the symmetric incomplete integration case. Suppose now that the timing of events is as follows. First, a NG signs a debt contract with international lenders that specifies the maximum amount, $\bar{b}_i$, the NG can borrow. We will suppose below that the level of indebtedness is smaller than the maximum (debt ceiling) for a given member NG so that $b = \phi\bar{b}$, where $0 < \phi < 1$ (where we drop subscripts for simplicity). Second, the NG undertakes public investment z_i. Next, the economy is hit by an unexpected negative productivity shock at the beginning of the second period such that $A' = \Psi A$, with $0 < \Psi < 1$. At this point, although levels of debt and investment cannot be changed, the decision to repay or default can be revisited. Solving the incentive constraints to avoid a default yields the condition for there to be repayment as

$$\Psi y_2 + (1-\delta)z - R\phi\bar{b} > (1-\eta)[\Psi y_2 + (1-\delta)z].$$

Recalling that $R\bar{b} = \eta[y_2 + (1-\delta)z]$, using equations (1) and (2), and solving this inequality for Ψ yields

$$\Psi > \phi - \frac{z^{1-\mu\beta(1+\theta)}}{A\Omega^{\beta(1+\theta)}}(1-\delta)(1-\phi) \tag{29}$$

Intuitively, this condition shows the largest negative productivity shock the economy can tolerate without leading to debt distress. Note that as the original debt level approaches the threshold level, that is, as ϕ increases, smaller (those with lower Ψ) negative shocks are sufficient to trigger debt distress. In (29) this is most clearly seen in the full depreciation case, $\delta = 1$.

Next, we investigate the resilience to shocks in a symmetric complete integration framework. Here the central government of the union allocates the funds that are raised by issuing debt evenly across the states when members are identical ex ante. Given the same sequence of events as above, we would like to find out how a central government that faces the same total debt ($2\phi\bar{b}$) as the NGs in the incomplete integration case will fare when hit by the same shock. As before repayment of the debt requires

$$\Psi^* y_2^* + (1-\delta)z^* - R\phi\bar{b} > (1-\eta)[\Psi^* y_2^* + (1-\delta)z^*]$$

where we assumed that the shock is also symmetric across member states. The counterpart to (29) is given by

$$\left(\frac{z^*}{z}\right)^{\mu\beta(1+\theta)} \Psi^* > \phi - \frac{(z^*)^{-\mu\beta(1+\theta)}}{A\Omega^{\beta(1+\theta)}} (1-\delta)(z^* - \phi z) \tag{30}$$

Given $z^* > z$, the left-hand side (right-hand side) of (30) is greater (smaller) than the left-hand side (right-hand side) of (29). Thus, shocks need to be greater in magnitude to trigger a default under complete integration.

Result 4: *Consider a union of symmetric countries with spillovers where unexpected productivity shocks hit the economy after debt contracts are signed and investment decisions are made. In this case, an incomplete integration scheme is less resilient to debt distress, i.e. shocks that would not trigger defaults under a complete integration scheme could trigger defaults if integration is incomplete.*

Finally, how would the two integration schemes differ when productivity shock hits only one state? With incomplete integration, the solution for the member state with smaller second period income is identical to (29) and for the other state there is no change as compared to initial conditions. Thus, the former defaults and the latter honors the debt contract if the shock is greater than the respective slack. In contrast, the central government of a union either defaults or honors the contract on the debt associated with investments in both regions simultaneously. Thus, suppose we start from symmetric conditions with slacks in both states. Consider a shock that would trigger default at the union level if it hits both states. Now, suppose the same magnitude of shock hits only one state. In this case, default could be avoided as the central government of the union would "transfer resilience" from one state to the other. This arises from the observation that defaulting has punishment as shares of incomes in *both* states, whereas the productivity shock reduced the income only in *one* state.

Finally, we will investigate the impact of shocks when the governments can change their investments upon learning about the productivity shocks.

5.2 Contagion of Shocks with Irreversible Debt and Flexible Investment

Here we are interested in exploring the potential for default and contagion when shocks were unexpected at the time of signing of international debt contracts but are observed

before investment decisions are made. Intuitively speaking, these shocks could be envisioned to be the kind that triggered the 2007 global financial crisis, which were thought to be low-probability events by the financial institutions involved. We are specifically interested in the consequences of such shocks for the provision of public infrastructure investment across member states in a union and their cross-border effects. It turns out that whether investments in member states are strategic complements or strategic substitutes have important consequences for understanding how productivity shocks affect the economy where they hit and how these shocks are transmitted to others via investment spillovers.

To fix ideas we start here with a symmetric Nash equilibrium in an incomplete integration scheme, *i.e.* given a predetermined level of debt, public investment in each country is a best response to the investment in the other one simultaneously. Suppose this equilibrium is then disturbed by an economy-wide shock in country-j that reduces the productivity levels for its second period output: $dA_j < 0$. The effect of this negative shock on public investment, z_j, in country-j is in general ambiguous. To see why recall our discussion above of the effects of changes in public investment in one country for public investment in the other country of the union and note that we face a similar question here. The negative supply shock $dA_j < 0$ on the one hand reduces the rate of return on investments and, thus, tends to reduce z_j, yet, on the other hand, by lowering future income, induces higher savings (through the consumption smoothing channel) and higher z_j. As discussed earlier, when $\mu\beta > 1$ the first mechanism dominates and public investment z_j falls when A_j decreases. As $\mu\beta > 1$ also implies that public investments are strategic complements, the reduction in z_j leads to a decrease in z_i as well.

Whether the negative shock will lead to default and, perhaps, contagion, depends on the magnitude of the shock and the extent of spillovers across countries. Table 1 provides some examples of supply shocks that lead in one case to default but not contagion but in another case to both default and contagion.[11] The case labeled "moderate shock" shows the effects of a negative supply shock that is relatively smaller on public investment levels and the payoffs to default and non default for both member countries of an economic union. In the case of incomplete integration, the table shows the $2^2 = 4$ combinations of investment decisions by two member country governments contemplating whether to repay or to default. In the simulation with a "moderate" shock, the country that is subjected to the shock defaults, but given the magnitude of the shock and the spillovers, the other

[11]The table is constructed by a choice of parameters that yield one member country 20 percent richer than the other one (in terms of initial income).

country chooses to repay its foreign debt.[12] When the magnitude of the negative shock is increased, even the (richer) member country that has been spared the shock chooses to default given the significant drop in its income through the decrease in the spillovers from the country that was subjected to the shock.

Table 1 also shows that a completely integrated union that has borrowed the same amount in international markets as the sum of the two union members of a union with incomplete integration can withstand the same output shocks without defaulting. One reason for this is the fact, discussed above, that the CG of an integrated union finds it optimal to invest more as it internalizes the externalities associated with public investment. Another reason, also discussed above, is that because the CG chooses to invest more, the debt ceiling it faces is higher than that of the sum of what the two countries face in a union without complete integration. One consequence of this is that when facing the same level of debt as the two countries, the CG would find itself to be under comparatively less debt stress when subjected to a negative shock. Thus, the table shows that neither the "moderate" shock that led to default by one country, nor the "bigger" shock that leads to contagion and, thus, default of both countries, would induce default by the CG. We summarize these findings in the following result.

Result 5: *Suppose public investments in different member economies are strategic complements. Then, there exists a range of idiosyncratic productivity shocks which: (i) trigger default in the economy subject to the shock and in the other economy via contagion when the integration is incomplete, and (ii) do not lead to either default or contagion in either economy when the integration is complete.*

6 Conclusions

This paper shows that the institutional design of fiscal integration has important implications for the ability to raise funds for public investments and stability in unions. Incomplete integration schemes fall behind complete integration schemes in both dimensions. This is mainly because national governments do not internalize the productivity enhancing spillovers between the jurisdictions. Thus, perceived returns to investments are smaller, and unilaterally optimal levels of investments are less than the one under complete integration.

[12]Note that $\partial U^i/\partial A_j = \theta\beta(\xi/c_{2i})(y_{2i}/z_j)(\partial z_j/\partial A_j)$ with θ and $\partial z_j/\partial A_j$ measuring the magnitude of the spillovers and the shock.

Table 1: Contagion of Debt Distress with Asymmetric Shocks

	Initial Equilibrium	Moderate Shock	Bigger Shock
A, A^*	4.0, 4.0	4.0, 2.0	4.0, 1.0
	Incomplete Integration		
z^N, z^{*N}	0.738, 0.639	0.738, 0.633	0.738, 0.626
z^N, z^{*D}	0.737, 0.620	0.737, 0.601	0.737, 0.582
z^D, z^{*N}	0.705, 0.639	0.704, 0.632	0.738, 0.625
z^D, z^{*D}	0.704, 0.618	0.703, 0.600	0.738, 0.626
$U(z^N, z^{*N}) \geq U(z^D, z^{*N})$	Yes	Yes	Yes
$U(z^N, z^{*D}) \geq U(z^D, z^{*D})$	Yes	Yes	No
$U^*(z^N, z^{*N}) \geq U^*(z^N, z^{*D})$	Yes	No	No
$U^*(z^D, z^{*N}) \geq U^*(z^D, z^{*D})$	Yes	No	No
	Complete Integration		
z^N, z^{*N}	0.845, 0.808	0.775, 0.775	0.772, 0.819
z^D, z^{*D}	0.831, 0.788	0.751, 0.751	0.738, 0.792
$U^{CG}(z^N, z^{*N}) > U^{CG}(z,^D z^{*D})$	Yes	Yes	Yes

Parameter Values: $\beta = 0.5, \Omega^{(*)} = 1$, $\mu = 3.0$, $\theta = 0.8$, $A = 2.0$, $R = 1.04$, $\xi = 0.9$, $y_1 = 1.2$, $y_1^* = 1.0$, $\delta = 0.05$, $\eta = 0.1$, $b = b^* = 0.13$, $b^{CG} = 0.26$

Notes:

Asymmetric regions. Member-2 (denoted by asterisks) receives negative productivity shocks. In the case of a moderate shock ($A^* = 2.0$) member-2 defaults, member-1 does not. With a bigger shock ($A^* = 1.0$), there is contagion: as member-2 defaults, this leads to a default by member-1 as well. In the case of a complete integration, with the same level of total foreign borrowing, the union chooses not to default when it faces the same negative productivity shocks.

We show that the above mentioned differences in investment behavior not only lead to different long-term income levels, but also imply different resilience to productivity shocks. First, the equilibrium income under a complete integration scheme is greater because the central government can borrow and invest more in a credible way. Second, there exists a range of idiosyncratic shocks which lead to default and contagion under incomplete integration but do not necessarily lead to either default or contagion under a complete integration scheme. The minimum size of a shock that leads to default and contagion is strictly greater under the latter system.

Our results inform recent policy discussions by showing how the institutional design of an economic union can make it more likely to exhibit defaults and contagion in the short-term and reduced income in the long-term. Moreover, we also provide support for establishing central authorities to perform fiscal coordination in issues with cross-jurisdiction implications. These findings are particularly relevant for the European Union, and the aspiring economic unions in East and West Africa, where emphasis has been on integration without fiscal centralization.

There are several promising extensions of the analytical framework developed in this paper. These include allowing factor mobility across member economies and endogenous formation of unions with variable numbers of members, which are left for future research.

References

[1] Alfaro, L., and Kanczuk, F. (2005): "Sovereign Debt as Contingent Claim: A Quantitative Approach," *Journal of International Economics*, 65, 297-314.

[2] Besley, T., and Coate, S. (2003): "Centralized versus Decentralized Provision of Local Public Goods: A Political Economy Approach," *Journal of Public Economics*, 87, 2611–2637.

[3] Buiter, W., Rahbari, E., and Seydl, J. (2014): "Secular Stagnation: Only If We Really Ask For It," Global Economics View, Citi Research.

[4] Bulow, J., and Rogoff. K. (1989): "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, 97 (1), 155-78.

[5] Cohen, J.P., and Morrison Paul, C.J. (2004): "Public Infrastructure Investment, Interstate Spatial Spillovers, and Manufacturing Costs," *The Review of Economics and Statistics*, Vol. 86, No. 2 (May, 2004), pp. 551-560.

[6] Fernandez, R., and Rosenthal, R.W. (1990): "Strategic Models of Sovereign-Debt Renegotiations," *Review of Economic Studies*, 57(3): 331-49.

[7] Glaeser, E.L. (2014): "Secular Joblessness," in Teulings, C. and Baldwin, R.(ed's), *Secular Stagnation: Facts, Causes and Cures*, CEPR Press.

[8] Gordon, R.J. (2012): "Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds," NBER Working Papers 18315, National Bureau of Economic Research, Inc

[9] Gordon, R.J. (2014): "The Demise of U.S. Economic Growth: Restatement, Rebuttal, and Reflections," NBER Working Papers 19895, National Bureau of Economic Research, Inc.

[10] IMF. (2014): "World Economic Outlook: October 2014," International Monetary Fund, Washington, DC.

[11] Jimeno, J.F., Smets, F., and Yiangou, J. (2014): "Secular Stagnation: A View From the Eurozone," in Teulings, C. and Baldwin, R.(ed's), *Secular Stagnation: Facts, Causes and Cures*, CEPR Press.

[12] Mendoza, E. C., and Yue, V.Z. (2008): "A Solution to the Default Risk-Business Cycle Disconnect," *National Bureau of Economic Research Working Paper,* No: 13861.

[13] Oates, W., 1972. Fiscal Federalism. Harcourt Brace, New York.

[14] Oates, W. (1999): "An Essay on Fiscal Federalism," *Journal of Economic Literature*, Vol. 37, No. 3 (September, 1999), pp. 1120-1149.

[15] Oates, W. (2005): "Towards a Second-Generation Theory of Fiscal Federalism," *International Tax and Public Finance*, Vol. 12, pp. 349-373.

[16] Rodden, J., 2005. Hamilton's Paradox: The Promise and Peril of Fiscal Federalism. Cambridge University Press, New York.

[17] Sachs, J., and Cohen, D. (1982): "LDC Borrowing with Default Risk," *National Bureau of Economic Research Working Papers,* No: 925.

[18] Sandleris, G. (2008): "Sovereign Defaults: Information, Investment, and Credit," *Journal of International Economics*, 76, 267-75.

Appendices

1. The Conditions for Strategic Substitutability or Complementarity of Investments in a Decentralized System

To find out under what conditions z_i and z_j are strategic substitutes or complements we need to know the sign of (10). Note first that it follows from (11) that the denominator of (10) is positive. Given the functional forms in (1) and (2) the numerator can be expressed as

$$\texttt{num} = Q_1 \left[\frac{\xi\mu\beta}{1+\xi\mu\beta} - \frac{z_i}{y_{1i}} \right] = -Q_2(1-\delta) \left[\frac{\xi}{1+\xi} - \frac{z_i}{y_{1i}} \right]$$

with

$$Q_1 \equiv \frac{(1+\xi\mu\beta)\theta\mu\beta y_{2i}}{y_1 z_i z_j} > 0, \; Q_2 \equiv \frac{(1+\xi)\theta\mu\beta y_1}{z_i z_j} > 0. \tag{31}$$

Thus, its sign depends on the sign of the expressions in square brackets in (31). These signs, in turn, can be determined by rearranging (8) to obtain

$$\left[\frac{\xi\mu\beta}{1+\xi\mu\beta} - \frac{z_i}{y_{1i}} \right] y_{2i} = -(1-\delta) \left[\frac{\xi}{1+\xi} - \frac{z_i}{y_{1i}} \right]. \tag{32}$$

(32) implies that for $y_{2i} > 0$, optimal public investment by an SNG is independent of the investment undertaken by its counterpart if $\delta = 1$:

$$z^{SNG} = \frac{\xi\mu\beta}{1+\xi\mu\beta} y_1.$$

On the other hand, if the rate of depreciation is less than a hundred percent, that is if $\delta < 1$, for $y_{2i} > 0$, (32) implies that

$$\frac{\xi\mu\beta}{1+\xi\mu\beta} < \frac{z_i}{y_{1i}} < \frac{\xi}{1+\xi} \; \texttt{if} \; \mu\beta < 1$$

and

$$\frac{\xi}{1+\xi} < \frac{z_i}{y_{1i}} < \frac{\xi\mu\beta}{1+\xi\mu\beta} \; \texttt{if} \; \mu\beta > 1.$$

Thus, public investments in infrastructure by the SNGs are strategic substitutes ($\partial z_i / \partial z_j < 0$) if $\mu\beta < 1$ and are strategic complements if $\mu\beta > 1$.